Curriculum Vitæ

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1 Personal

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1.1 Employment history

SISSA/ISAS	**"Professore di I fascia" (full professor, at part time)** at "Scuola Internazionale Superiore di Studi Avanzati" (Trieste, Italy) from January 2015 ("a tempo definito"). Currently on Leave of Absence.
Concordia University	**Full Professor** (tenured) at Concordia University (Montréal) from June 2013
Concordia University	**Associate Professor** (tenured) at Concordia University (Montréal) from June 2007 to June 2013.
Concordia University	**Assistant Professor** (tenure-track) at Concordia University (Montréal) from August 2002 to May 2007.
Concordia University	**Research Professor** at Concordia University (Montréal) from January 2000 to April 2002

1.2 Academic background

CRM-ISM	**Postdoctoral fellowship** from October 1999 to April 2002 at the *Centre de recherches mathématiques (CRM)*, Université de Montréal.
SISSA, Trieste	October 1995, September 1999: Admission to the Ph.D. program in Mathematical Physics at SISSA–ISAS, in Trieste. **Ph.D. Thesis** defended on September 4, 1999 with the title "Jacobi groups, Jacobi Forms and their Applications", under the supervision of Prof. Boris Dubrovin.
University of Milan	22 March 1995: *Laurea* **(Degree) in Physics** with full marks and honors ("110/110 con Lode") Dissertation: *"Effetti termici della quantizzazione in uno spazio–tempo curvo"* (Thermal Effects of Quantization on curved spacetimes). Supervisors: Prof. V. Gorini, Dr. Mauro Zeni.

Awards • Dean's Award for Distinguished Scholarship, Concordia University 2011.

2 Research

2.1 Research Statement

Over the course of my career, my interests have been quite diverse within the general area of mathematical physics. I will describe the active ones in the paragraphs to follow, with reference to the corresponding publications. The general theme and tools straddle the boundary between (complex) analysis, spectral theory, (algebraic/symplectic) geometry and special functions.

In the sections below, for each area of research, I will describe some representatives of the results that my collaborators and myself have obtained. Inevitably, the selection of examples is only a very reductive illustration of the breadth of results, and it reflects my own personal preference.

2.1.1 Symplectic geometry of moduli spaces and character varieties

[1, 2, 7, 9, 12, 19, 24, 26] This is the most recent direction amongst my interests. In general it revolves around the notion of *monodromy map* to be intended in a wide sense. The classical (Schlesinger, Gambier) definition involves the $n \times n$ matrix solution of a system of differential equations (Cauchy Initial Value Problem) in the complex plane with Fuchsian singularities of the form:

$$\Psi'(z) = A(z)\Psi(z), \quad \Psi(z_0) = \mathbf{1}, \qquad A(z) := \sum_{j=1}^{K} \frac{A_j}{z - t_j}. \tag{1}$$

Analytic continuation of Ψ along a (homotopy class of) path $\gamma \in \pi_1(\mathbb{P}^1 \setminus \{\vec{t}\}, z_0)$ yields a map into the character variety:

$$M \in Hom(\pi_1, \mathrm{GL}_n)//\mathrm{GL}_n = \mathcal{M} \tag{2}$$

Both the space,$\mathcal{A}$, of matrices $A(z)$ and the character variety, $\mathcal{M}$, are Poisson manifolds with the Kostant-Kirillov-Souriau (i.e. Lie–Poisson) structure on the side of $\mathcal{A}$ and the Goldman symplectic structure on $\mathcal{M}$. The monodromy map is known to be a Poisson map [Korotkin-Samtleben '96, Hitchin '97, Jeffrey '94, Boalch '00-'07]. One can define suitable fibrations over both $\mathcal{A}$ and $\mathcal{M}$ in such a way that the Poisson structures extend to symplectic structures. The main philosophy of [1] is that the polarizations of these two symplectic structures induced by the KKS symplectic potential Θ_{KKS} on the side of $\mathcal{A}$ and a suitable local symplectic potential θ_{FG} for the (extended) Goldman structure on the $\mathcal{M}$ side (defined *explicitly* in terms of the Fock—Goncharov coordinatization of the character variety) determines a "function" (technically only the section of a line–bundle) by $\mathrm{d}\ln\tau = \Theta_{KKS} - \theta_{FG} - \sum_{j=1}^{K} H_j \mathrm{d}t_j$; here $H_j = \frac{1}{2}\operatorname{res}_{z=t_j} \operatorname{Tr} A(z)^2 \mathrm{d}z$. When restricted to the submanifold where the monodromy representation is fixed, this function turns out to coincide with the "isomonodromic tau function" defined in the eighties by the Japanese school of Jimbo, Miwa, Ueno, Sato. Thus the above formula gives a concrete way to define the dependence on the monodromy data of the "isomonodromic" tau function, which possibly sounds like an oxymoron.

Closely related to this work is [2] where, using an idea developed in [11], we define the *canonical two–form* associated to a flat connection on an embedded ribbon graph. Given a (isotopy class of) graph Σ on a Riemann surface and a group–valued function $J : \mathbf{E}(\Sigma) \to G$ (here $\mathbf{E}$ denotes the set of oriented edges), satisfying a suitable no-monodromy condition at each vertex, then one define a two form by the formula

$$\Omega(\Sigma) = \sum_{v \in \mathbf{V}(\Sigma)} \sum_{\ell=1}^{n_v - 1} \operatorname{Tr}\left((J^{(v)}_{[1:\ell]})^{-1} \mathrm{d}J^{(v)}_{[1:\ell]} \wedge (J^{(v)}_{\ell})^{-1} \mathrm{d}J^{(v)}_{\ell} \right) \tag{3}$$

where $J^{(v)}_\ell$ denote the matrices on the edges incident to $v \in \mathbf{V}(\Sigma)$ and the subscript $_{[1:\ell]}$ denotes the product of the matrices associated to the incident edges (enumerated counterclockwise). This form is shown to be closed and it is symplectic under some special circumstances. If the graph/jump matrices are chosen appropriately, it represents the (extended) Goldman symplectic structure. For $G = SL_n$ we explicitly parametrize these matrices (which can be used to define a point of the character variety). A major result is that in these coordinates the two form becomes *log-canonical* (i.e. it has constant coefficients in the logarithms of the coordinates). This has deeply rooted connections with the theory of Cluster Algebras and opens the path to a concrete description of the line–bundle whose Chern class is represented by Ω above (up to a multiplicative constant).

Along similar lines are the results of [26]; in this case we are discussing the space of *holomorphic opers*, namely, ODEs of second order on a Riemann surface in the form

$$(\partial_z^2 + \frac{1}{2}\mathfrak{U})\varphi = 0 \tag{4}$$

where $\mathfrak{U}$ is a holomorphic projective connection, i.e., an object described locally by a holomorphic function $U(z)$ that transforms under change of coordinates according to the affine rule (involving the Schwartzian derivative of the change of coordinates)

$$U(z)\mathrm{d}z^2 = \widetilde{U}(w)\mathrm{d}w^2 + \left(\frac{\mathrm{d}}{\mathrm{d}w}\left(\frac{\mathrm{d}^2 z}{\mathrm{d}w^2}\right) - \frac{1}{2}\left(\frac{\mathrm{d}^2 z}{\mathrm{d}w^2}\right)^2\right)\mathrm{d}w^2. \tag{5}$$

The dependent variable φ must transform as a $-\frac{1}{2}$ differential for the equation to have invariant meaning (i.e. independent of the coordinate).

The space, $\mathbb{P}\mathrm{roj}$, of such projective connections is an affine space modelled on the linear space of quadratic (holomorphic) differentials, $\mathcal{Q}_g[\mathcal{C}]$, on the Riemann surface $\mathcal{C}$. This space is identified with the co-tangent space of the moduli (Teichmüller) space at $\mathcal{C}$. Taking this description in families, we see that $\mathcal{Q}_g \simeq T^*\mathcal{M}_g$ (where $\mathcal{M}_g$ denotes rather the moduli space). The identification between $\mathcal{Q}_g$ and $T^*\mathcal{M}_g$ needs a choice of a "reference" holomorphic projective connection that also depends holomorphically on the moduli of the curve. The main result, conceptually, of loc. cit. is depicted in the following diagram, where Pexp denotes the monodromy map, V denotes the character variety and V_{ne} denotes the open-dense set of non-elementary representation, which, from a result of [Gallo-Kapovich-Marsden], is a local epimorphism:

$$\begin{tikzcd}
\mathcal{Q}_g \simeq T^\star\mathcal{M}_g \arrow[r, leftrightarrow, "\mathfrak{U}_0+\mathcal{Q}_g"] \arrow[dr] & \mathbb{P}\mathrm{roj} \arrow[r, twoheadrightarrow, "\mathrm{Pexp}"] \arrow[d] & V_{ne} \subset V \\
& \mathcal{M}_g &
\end{tikzcd} \tag{6}$$

Now clearly $T^*\mathcal{M}_g$ (as any co-tangent space) is a (complex) symplectic manifold, and so is the character variety V (with the Goldman symplectic structure). The main result can then be formulated in this "meta" theorem

Theorem: the map Pexp is Poisson, if and only if the base projective connection $\mathfrak{U}_0$ is in the equivalence class of the Bergman projective connection.

The "equivalence" we are alluding to here is that two (local) holomorphic sections of the fibration $\mathbb{P}\mathrm{roj} \to \mathcal{M}_g$ are "equivalent" if their difference is (identifiable with) the differential of a local holomorphic function on $\mathcal{M}_g$.

This result reprises results of [Kawai, S., Math Ann **305** 161-182 (1996)] where the author used a completely different approach based on Bers' quasi-Fuchsian uniformization.

2.1.2 Inverse problems and spectral theory

[3, 4, 20, 34, 36, 50]

This is the line of research where the techniques of integrable systems have been applied to various problems related to stability analysis of tomographic reconstruction, and area where my collaborator A. Katsevich (UCF) is a leading expert. To make an example of the type of problems in this direction let me mention

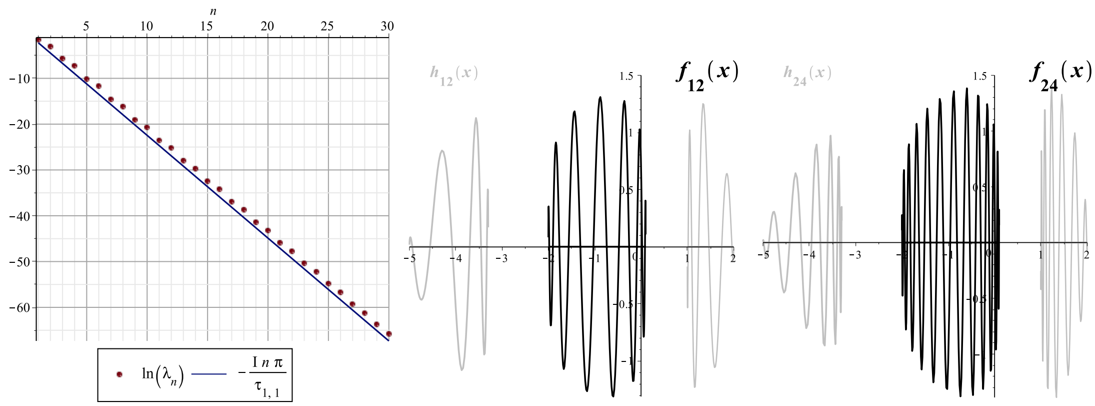

Figure 1: Plotted in the figure are $\ln(\lambda_n)$ (the eigenvalues of $\mathscr{K}$). The straigh line's slope is predicted in terms of the geometry of the intervals E, J. The small fluctuations of λ_n away from linear behaviour are explained by the periodicity of the Theta divisor in a suitable Jacobian. The eigenvalues, asymptotically, are in correspondence with the intersections of a real line with the Theta divisor in a (real) Jacobian. Also plotted are two pairs of the corresponding singular functions (f_{12}, h_{12}) and (f_{24}, h_{24}), obtained numerically simultaneously with λ_n. The envelope of the oscillations is already visibly the same and can be described analytically in the paper.

the latest [3]. Here we study the spectral properties of a classical operator in complex analysis. Consider $J, E \subset \mathbb{R}$ two closed sets (each a finite union of intervals) with disjoint interior and the operator

$$A : L^2(J) \to L^2(E), \ (Af)(x) = \frac{1}{\pi} \int_J \frac{f(y)\mathrm{d}y}{x - y},$$

and let $A^\dagger$ be its adjoint. The issue at hand here is the analysis of the singular spectrum, or equivalently, the spectral properties of the self-adjoint operator $\mathscr{K}$ acting on $L^2(E) \oplus L^2(J)$, whose off-diagonal blocks consist of A and $A^\dagger$.

The spectral properties of the operator discussed here is at the core of the issue of stability in the reconstruction of tomographic images using the [Gel'fand–Graev '91] formula. Here I quote one of the several results that we have obtained in this area (from [3]) to illustrate the tenor of this research.

Theorem 1. *One has:*

1. $\mathfrak{Sp}(\mathscr{K}) \subseteq [-1, 1]$;
2. *There is an absolutely continuous component* $\mathfrak{Sp}_{ac}(\mathscr{K}) = [-1, 1]$ *of* $\mathfrak{Sp}(\mathscr{K})$ *if and only if there is a double point. Moreover, the multiplicity of* $\mathfrak{Sp}_{ac}(\mathscr{K})$ *is equal to the number of double points (i.e. points in* $E \cap J$*);*
3. *The end points* $\lambda = \pm 1$ *of the spectrum* $[-1, 1]$*, as well as* $\lambda = 0$*, are not eigenvalues. Moreover,* $\mathscr{K}$ *is of trace class if and only there are no double points. In this case,* $\mathfrak{Sp}(\mathscr{K})$ *consists only of eigenvalues and* $\lambda = 0$*, which is the accumulation point of the eigenvalues;*
4. *The eigenvalues of* $\mathscr{K}$ *are symmetric with respect to* $\lambda = 0$ *and have finite multiplicities. Moreover, they can accumulate only at* $\lambda = 0$*;*
5. *The singular continuous component is empty, i.e.,* $\mathfrak{Sp}_{sc}(\mathscr{K}) = \varnothing$.

If E and J have positive relative distance, then one can find asymptotic description (under some additional assumption) of the eigenvalues. I will only illustrate this by the Fig. 1 from [34], with some explanation in the caption.

The problem is closely related to the analysis of the spectral properties of the Hilbert transform on a finite interval [Okada-Elliot '91] and it is only surprising that the above theorem is apparently a novel result for an operator that is at the root of so many inverse and boundary problems. The interesting and novel approach is the use of a suitable boundary value problem for a matrix–valued function (a.k.a. a *Riemann Hilbert problem*, see below) which is precisely where my expertise and the techniques of integrable systems play a crucial role.

2.1.3 Random Matrices and Random Point Fields

[85, 84, 83, 82, 81, 86, 80, 79, 77, 76, 75, 74, 70, 69, 67, 65, 64, 63, 62, 59, 58, 51, 49, 45 56, 53, 47, 40, 37]

The spectral theory of random matrices originated in the work of [Wigner '51] on the spectra of heavy nuclei. It was then developed by Mehta, Gaudin, Dyson and found many applications in solid state physics, 2D-quantum gravity and string theory. In the context of integrable systems it is known that matrix integrals give special realizations of KP, Toda and isomonodromic τ-functions, which can be thought of as the ultimate generalization of the Riemann theta functions (which are themselves a generalization of the trigonometric functions), satisfying a plethora of functional relations.

Multi–matrix models are a generalization of one–matrix models and have been studied by large number of people like [Itzykson-Zuber '80], [Mehta '81], [Daul-Kazakov-Kostov '93], [Mehta-Shukla '81], [Adler-VanMoerbeke '97-'99] to name just the first few.

In general they can be presented as a probability measure on some ensemble of matrices, for example, Hermitean matrices of size $N \times N$. The types of probability measures most related to my research are of the form

$$\mathrm{d}\mu(M_1, \ldots, M_R) = \frac{1}{\tau} \prod_{j=1}^{R} \mathrm{d}M_j \mathrm{e}^{-\Lambda \mathrm{tr} V_j(M_j)} \prod_{k=1}^{R-1} I(M_k, M_{k+1}) \tag{7}$$

where the coupling terms $I(M, M')$ are of some particular form, which determines the properties of the statistics. Here $\mathrm{d}M_j$ is the standard Lebesgue measure on the space of Hermitean matrices and V_j (the *potentials*) are scalar functions (defined on the Hermitean matrices via the spectral theorem).

The issues of main interest are related to the statistics of the eigenvalues of the matrices, especially in the limit as the size N tends to infinity and the potentials V_j are scaled so that the multiplicative constant Λ grows as $\mathcal{O}(N)$.

The case $R = 1$ constitutes the more studied one-matrix model, while the multimatrix models ($R \geq 2$) display properties that depend on the coupling factor $I(M, \widetilde{M})$ in the measure (7);

- $I(M, \widetilde{M}) = \mathrm{e}^{N \mathrm{tr} M \widetilde{M}}$ is the more common form of interaction (*Itzykson–Zuber* or *Harish-Chandra* interaction);
- the *Cauchy interaction* $I(M, \widetilde{M}) = \det(M + \widetilde{M})^{-N}$ [58, 63]

The *partition function* τ, namely, the normalizing factor in the probability measure depends on the potentials according to some integrable hierarchy, specifically

- $R = 1$: Toda lattice and Kadomtsev–Petviashvili hierarchies;
- $R \geq 2$: 2Toda lattice and multicomponent KP [Ueno-Takasaki '84], [Adler-VanMoerbeke '97–'99]

(Multi)-orthogonal polynomials. In all the cases, the joint probability distributions (*jpdf*'s) of the eigenvalues can be expressed in terms of *determinants*, so that the eigenvalues undergo what is commonly referred to as a *determinantal point process*

$$\mathcal{R}(\vec{x}_1, \ldots \vec{x}_R) = \det[K_N^{(i,j)}(\vec{x}_i, \vec{x}_j)]_{i,j \leq R}. \tag{8}$$

It turns out that the *kernels* $K^{(i,j)}(x,y)$ can be expressed in terms of (multi)orthogonal polynomials or certain integrals transforms thereof.

I will describe in some detail the simplest case ($R=1$) but analogous threads (with some important complications) can be followed in the other models. In the case of a model with measure $\mathrm{d}M \mathrm{e}^{-\Lambda \mathrm{tr}\, V(M)}$, the kernel is expressed as

$$K_N(x,x')= \sum_{j=0}^{N-1} \frac{p_j(x)p_j(x')}{h_j} \mathrm{e}^{-\frac{\Lambda}{2}\left(V(x)+V(x')\right)} \tag{9}$$

$$\int_{\mathbb{R}} p_n(x)p_m(x)\mathrm{e}^{-\Lambda V(x)}\mathrm{d}x = h_n\delta_{mn}\ . \tag{10}$$

where the polynomials $p_n(x)$ are the (monic) orthogonal polynomials for the measure $\mathrm{e}^{-\Lambda V(x)}\mathrm{d}x$.

The important technical tool is that the orthogonal polynomials can be described with the help of a (non-Abelian) Riemann–Hilbert problem , as well as other multi-orthogonal polynomials. In the specific case the problem is that of finding a 2×2 matrix-valued function $\Gamma(z)$, holomorphic in $\mathbb{C}\setminus\mathbb{R}$ and admitting the boundary values

$$\Gamma_n(z)_+ = \Gamma_n(z)_- \left[\begin{array}{cc} 1 & \mathrm{e}^{-\Lambda V(z)} \\ 0 & 1 \end{array}\right] ,\quad z\in\mathbb{R} \tag{11}$$

$$\Gamma_n(z) \sim \left(\mathbf{1}+\mathcal{O}(z^{-1})\right) \left[\begin{array}{cc} z^n & 0 \\ 0 & z^{-n} \end{array}\right] \tag{12}$$

The orthogonal polynomial $p_n(x)$ is then the $(1,1)$ matrix entry of the solution and additionally the kernel is given by $K_N(x,x') = \frac{\left[\Gamma_N^{-1}(x)\Gamma_N(x')\right]_{21}}{2i\pi(x-x')}$.

Asymptotic analysis in nonlinear PDEs and orthogonal polynomials. The formulation in terms of a Riemann–Hilbert problem is essential in the study of the statistical spectral properties in the asymptotic regime $N\to\infty,\ \Lambda=\mathcal{O}(N)$ using the *non-Abelian steepest descent method* developed originally by Deift and Zhou and elaborated in [Deift-Kriecherbauer-McLaughlin-Venakides-Zhou '99]

Without dwelling upon the method we point out that a major component calls for the introduction of a special scalar function (generally called g–function in the pertinent literature) which must solve certain specific problems in *potential theory*; typically the real part of g is harmonic away from certain contours and with certain inequalities being satisfied. While for ordinary orthogonal polynomials the contour consists of (subintervals of) the real axis, we did study more general situations (*semiclassical orthogonal polynomials*) where the potential $V(z)$ is complex valued and the contour of integration defining the (nonHermitean) orthogonality is chosen as collection of paths in the complex plane. In this case the construction of the g–function becomes a difficult problem on its own (see [67, 55]).

Random point processes and Fredholm determinants The study of "gap probabilities" of Random Point processes of determinantal type (DRPP), namely probability of finding no points in a given subset, is deeply linked with the theory of Fredholm determinants. In special cases, since the works of [Tracy-Widom '94-06] it has become clear that these determinants can be expressed in terms of special solutions of Painlevé equations (and the like of it). Since these gap probabilities depend on parameters, it is of interest to investigate their asymptotics in special regime of growth of their parameters. Their formulation in terms of Riemann–Hilbert problems is the first step in this direction.

2.1.4 Orthogonal functions, integrable systems and nonlinear waves

[6, 5,22, 25, 28, 31, 32, 35, 38, 41, 42, 43, 44, 46,48, 52, 54, 55, 57, 61, 67, 68, 71, 72]

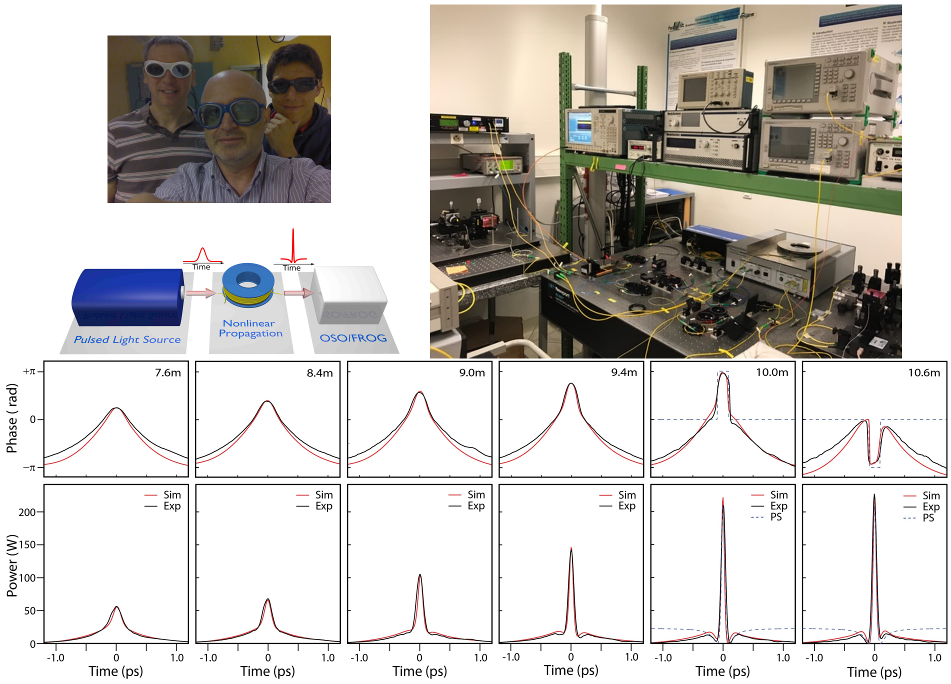

Figure 3: Experiment (black line) and NLS simulations (red line). Shown also the lab setup (and the brave collaborators that work in the lab. Cool glasses!) where the predictions from the theoretical work [46] were tested with pulsed lasers.

Universality in integrable nonlinear PDEs. An example of the type of problems we study in this area is the focusing Nonlinear Schrödinger (NLS) equation,

$$i\hbar\partial_t q = -\hbar^2\partial_x^2 q - 2|q|^2 q \tag{13}$$

$$q(x,0,\hbar) = A(x)e^{i\Phi(x)/\hbar} \tag{14}$$

which models self-focusing and self-modulation (*optical fibers*). It is *integrable* by inverse scattering methods (Zakharov–Shabat). We study $\hbar \to 0$ (a.k.a. the "dispersionless limit") for wide classes of initial data and try to describe qualitatively and quantitatively the asymptotic behaviour of the solution. The problem here is to describe the transition of the small dispersion wave from a region (in space-time) with a smooth modulated behavior to a region displaying paroxysmal oscillations. In [46] we showed that the behavior is "universal" in the sense that it depends on the initial data only trough a finite number (typically small) of constants (that do depend on the initial datum) and otherwise is described by general formulæ (see Fig. 2).

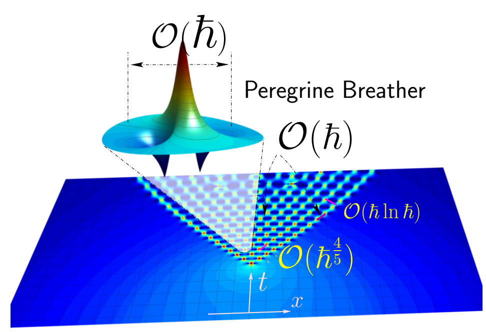

Figure 2: The amplitude (colormap) $|q(x,t)|$ for small $\hbar$ (numerical simulation); indicated the various scales and enlarged the Peregrine breather structure near the gradient catastrophe point (the vertex of the region of paroxysmal oscillations).

These results were, to my surprise (but sometimes math has practical applications!) verified experimentally in a work [25]. See Fig. 3.

The main result of [46] is that near the "gradient catastrophe", the pulsed structures have a universal profile in the form of the so–called "Peregrine breather"; in particular, the maximal amplitude of the oscillations *does not diverge* (as $\hbar \to 0_+$) but has a universal limit gain of 3 over the average background amplitude.

(Bi)-Orthogonal functions and integrable systems. The theory of orthogonal polynomials and similar notions is tied to several branches of mathematics; approximation theory, analysis, integrable systems, random matrices and combinatorics. My interest started because of their appearance in the theory of random matrix ensembles and evolved to encompass the aspects of their asymptotic analysis. To remind of the context, in their simplest incarnation (non-hermitean) orthogonal polynomials are a sequence of (monic) polynomials $P_n(z)$ of degree n such that

$$\int_{\mathbb{R}} P_n(z)P_m(z)\mathrm{d}\mu(z) = h_n\delta_{nm}, \tag{15}$$

for some density (measure, possibly complex-valued) $\mathrm{d}\mu$. The notion has well-known applications to physics as eigenfunctions of some Sturm–Liouville operators appearing in quantum mechanics. Their origin, however, lies in the theory of Padé approximations. The connection with integrable systems (and Painlevé equations) starts if we let $\mathrm{d}\mu(z;t)$ depend on "time" as $\mathrm{d}\mu(z;t) = \mathrm{e}^{zt}\mathrm{d}\mu(z)$. In that case the recurrence coefficients of the OPs provide a solution to the so-called Toda-lattice equations.

Very recently I have introduced a new notion of orthogonality that stems from the generalization to higher genus Riemann surfaces, of the classical Padé approximation problem. In the classical case where $\mathcal{C} = \mathbb{P}^1$ the problem consists in approximating the Weyl–Stieltjes function of the measure:

$$W(z) := \int_{\mathbb{R}} \frac{\mathrm{d}\mu(x)}{z-x} \tag{16}$$

by a ratio of polynomials Q_{n-1} and P_n of the indicated degrees, so that the difference satisfies:

$$\frac{Q_{n-1}(z)}{P_n(z)} - W(z) = \mathcal{O}(z^{-(2n+1)}). \tag{17}$$

The denominators, P_n, of this Padé approximation problem are famously the orthogonal polynomials introduced above. The question of how to generalize this problem to a Riemann surface other than $\mathbb{P}^1$ faces the first obstacle in the proper generalization of $\frac{1}{z-x}$: one must use, instead, a suitably defined Cauchy kernel, which depends parametrically on a divisor $\mathscr{D}$ of degree g. Then the polynomials are replaced by sections of $\mathcal{O}(n\infty + \mathscr{D})$ (where ∞ is a chosen point on $\mathcal{C}$) and they turn out to also satisfy an orthogonality like the ordinary polynomial case.

What is also particularly interesting is that one can associate a matrix Riemann–Hilbert problem to the Padé approximation scheme and this can be used effectively to compute asymptotic behaviours of these orthogonal sections as their degree tends to infinity.

Alternatively [103] one can orthogonalize section of two "dual" line bundles

$$\mathscr{P}_n := H^0(\mathcal{X} \otimes \sqrt{\mathcal{K}}((n+1)\infty)), \quad \mathscr{P}_n^\vee := H^0(\mathcal{X}^\vee \otimes \sqrt{\mathcal{K}}((n+1)\infty)), \tag{18}$$

where $\mathcal{X}$ is a flat line-bundle and the pairing is given in terms of integration along a path $\gamma \subset \mathcal{C}$ with a density function $w : \gamma \to \mathbb{C}$ on it:

$$\langle \varphi, \varphi^\vee \rangle = \int_\gamma \varphi\varphi^\vee w, \quad \varphi \in \bigoplus \mathscr{P}_n, \quad \varphi^\vee \in \bigoplus \mathscr{P}_n^\vee. \tag{19}$$

Either of the two approaches gives rise to a 2×2 Riemann–Hilbert problem that determines a vector bundle of degree $2g$ over $\mathcal{C}$. We have already shown in the recent preprint [104] how this can be used effectively to determine asymptotic behaviours of the orthogonal sections. This is a fresh line of research that I consider to hold extremely high potential impact in both the approximation theory community as well as the integrable-systems one.

Integrable systems approaches to enumerative geometry. [10, 8, 14, 15, 17, 18, 23, 30, 33 ,39] There is an intimate connection between the theory of integrable systems and moduli spaces of Riemann surfaces which was first conjectured by [Witten '90] and shortly after proved (almost rigorously) by Kontsevich. Namely, one defines the "intersection numbers"

$$\left\langle \tau_0^{k_0}\tau_1^{k_1}\dots\right\rangle := \int_{\overline{\mathcal{M}}_{g,n}} \psi_1^{\ell_1}\wedge\dots\wedge\psi_n^{\ell_n} \tag{20}$$

$$k_j = \sharp \text{ of times } j \text{ appears as exponent} \tag{21}$$

where $\mathcal{M}_{g,n}$ denotes the moduli space of stable Riemann surfaces with n marked points and ψ_j denote the Chern classes of the tautological line bundles. The conjecture/theorem alluded to above is that if we arrange those numbers into the (formal) series

$$F(t_0,t_1,\dots,) := \sum_{k_1,k_2,\dots} \left\langle \tau_0^{k_0}\tau_1^{k_1}\dots\tau_\ell^{k_\ell}\dots\right\rangle_{\substack{\overline{\mathcal{M}}_{g,n}\\ n=\sum k_j,\\ 3g-3=\sum(j-1)k_j}} \prod_{j=0}^{\infty}\frac{t_j^{k_j}}{k_j!} \tag{22}$$

we obtain a (formal) solution of the KdV hierarchy. Namely the function

$$U(t_0,t_1,\dots) := \frac{\partial^2 F(\vec{t})}{\partial t_0^2} \tag{23}$$

solves

$$\left\{\begin{array}{l} \dfrac{\partial U}{\partial t_1} = U\dfrac{\partial U}{\partial t_0} + \dfrac{1}{12}\dfrac{\partial^3 U}{\partial t_0^3} \quad \text{(Korteweg-deVries equation)}\\ U(t_0,0,\dots,) = t_0, \end{array}\right. \tag{24}$$

as well as infinitely many higher order PDEs (which constitute the aforementioned KdV hierarchy). The key tool introduced by Kontsevich was the following *matrix integral*

$$Z_n(x;Y) := \frac{\displaystyle\int_{H_n} \mathrm{d}M \mathrm{e}^{\mathrm{Tr}\left(i\frac{M^3}{3}-YM^2+ixM\right)}}{\int_{H_n}\mathrm{d}M\mathrm{e}^{-\mathrm{Tr}\,(YM^2)}}, \tag{25}$$

where $Y=\mathrm{diag}(y_1,\dots,y_n)$ and H_n denotes the vector space of Hermitean matrices of size $n\times n$; as $n\to\infty$ the expansion of the integral for $Y\to\infty$ in the symmetric polynomials $t_j=-(2j-1)!!\mathrm{Tr}\,(Y^{-2j-1})$ of the inverses of the variables y_j reproduces, up to order n, the formal series of Witten.

This integral is therefore a way to investigate the *analytic* properties of Borel resummations of the formal series [30] and clarify its source as an isomonodromic tau function of an appropriate isomonodromic system (which I don't describe here). In short the result here is that the formal series (22) is actually the asymptotic expansion, in suitable poly-sectors, of honest analytic functions of the times $t_0,\dots$. This is the only result I am aware of that addresses the actual analytic convergence of the formal manipulations in the mathematical physics literature.

Similar techniques were then extended and applied to other similar matrix integrals that appear to generate Gromov-Witten invariants [10, 15] and the intersection numbers for moduli spaces of open Riemann surfaces [18]. One of the most important results in this area is contained in [33] and consists in the explicit formula (non-recursive) for the generating functions of the intersection numbers in $\mathcal{M}_{g,n}$.

Theorem 2. *Let $M(z)$ denote the following matrix-valued formal series*

$$M(z) = \frac{1}{2}\begin{pmatrix} -\sum_{g=1}^{\infty}\frac{(6g-5)!!}{24^{g-1}\cdot(g-1)!}z^{-6g+4} & -2\sum_{g=0}^{\infty}\frac{(6g-1)!!}{24^g\cdot g!}z^{-6g} \\ 2\sum_{g=0}^{\infty}\frac{6g+1}{6g-1}\frac{(6g-1)!!}{24^g\cdot g!}z^{-6g+2} & \sum_{g=1}^{\infty}\frac{(6g-5)!!}{24^{g-1}\cdot(g-1)!}z^{-6g+4} \end{pmatrix}. \tag{26}$$

The generating functions of n-point intersection numbers

$$F_n^{WK}(z_1,\dots,z_n) := \sum_{k_1,\dots,k_n=0}^{\infty} \langle \tau_{k_1}\dots\tau_{k_n}\rangle \frac{(2k_1+1)!!}{z_1^{2k_1+2}}\cdots\frac{(2k_n+1)!!}{z_n^{2k_n+2}}, \quad n\geq 1 \tag{27}$$

are given by the following formulæ:

$$F_1^{WK}(z) = \sum_{g=1}^{\infty} \frac{(6g-3)!!}{24^g\cdot g!} z^{-(6g-2)}, \tag{28}$$

$$F_n^{WK}(z_1,\dots,z_n) = -\frac{1}{n}\sum_{r\in S_n} \frac{\mathrm{Tr}\left(M(z_{r_1})\cdots M(z_{r_n})\right)}{\prod_{j=1}^{n}(z_{r_j}^2 - z_{r_{j+1}}^2)} - \delta_{n,2}\frac{z_1^2+z_2^2}{(z_1^2-z_2^2)^2}, \quad n\geq 2. \tag{29}$$

A common thread: Riemann–Hilbert problems. A common thread to many of the previous lines of research is the connection, in various guises, to the notion of a Riemann Hilbert Problem (RHP).

To explain it in simple terms we consider the prototypical example of finding a matrix–valued, piecewise analytic function $\Gamma(z)$ with a discontinuity across an oriented path γ (e.g. the unit circle) and satisfying the boundary-value conditions:

$$\Gamma(z_+) = \Gamma(z_-)J(z), z\in\gamma \quad \Gamma(\infty) = \mathbf{1}, \tag{30}$$

where the subscripts $\pm$ denote the boundary values from the left/right (respectively) on γ.

If the "jump matrix" is analytic in an annulus, this problem can be equivalently presented as finding a trivialization of a vector bundle on the Riemann sphere with J as transition function in the overlap of two open disks.

If $J(z;\vec{t})$ depends (analytically) on additional parameters, one can define the so–called "Malgrange one–form" [60, 11]

$$\Theta := \int_\gamma \mathrm{Tr}\left(\Gamma_-^{-1}\frac{\mathrm{d}\Gamma_-}{\mathrm{d}z}\delta J J^{-1}\right)\frac{\mathrm{d}z}{2i\pi} \tag{31}$$

where δ represents the total differential in the additional parameters $\vec{t}$. The form was implicitly contained in works of Malgrange and proves to be related, in several instances to the differential of a Fredholm determinant.

It is quite astonishing that many, if not all, of the problems described above can be reduced to the analysis of a RHP of similar nature, but it is beyond the scope of this brief description.

Here I will mention the recent result [101] where we generalize the notion of Malgrange form to RHP on higher genus Riemann surfaces. The key object of the construction is the non-Abelian Cauchy kernel. This is a matrix–valued kernel depending on two points $p,q\in\mathcal{C}$ that is a meromorphic differential in p and a meromorphic function in q. It can be *explicitly* constructed in terms of the *Tyurin data* [Tyurin '65], which are a very concrete parametrization of the moduli space of vector bundles of rank n and degree ng.

The Tyurin data are (generically) a divisor $\mathscr{T}$ of degree ng and a point $[\mathbf{h}_j]\in\mathbb{P}^n$ for each of the points of $p_j\in\mathscr{T},\ j=1,\dots ng$. In terms of these data there exists a unique $n\times n$ *(matrix) Cauchy kernel* $\mathbf{C}_\infty(q,p)$ such that

- it is a meromorphic differential in the variable q with its divisor satisfying $(\mathbf{C}_\infty(q,p))_q \geq -p-\infty$;
- it is a meromorphic function in the variable p with its divisor satisfying $(\mathbf{C}_\infty(q,p))_p \geq \infty - q - \mathscr{T}$,

and such that $\mathbf{h}_j\mathbf{C}_\infty(p_j,p) = 0,\ \forall p_j\in\mathscr{T}$, together with the normalization that the residue along the diagonal is the identity. The Tyurin data determine uniquely a vector bundle $\mathscr{E}$ and the Cauchy kernel exists and is unique if and only if $h^1(\mathscr{E}) = 0$ (i.e. $h^0(\mathscr{E}) = n$, by the Riemann–Roch). Namely it is defined outside of the *non-abelian Theta divisor*, which is the locus in the moduli space where $h^0(\mathscr{E}) > n$.

Without entering into details we show this (simple) result involving the Tyurin data. Define $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{ng}] \in \mathrm{Mat}_{n,ng}(\mathbb{C})$ (the normalization of the Tyurin vectors are chosen arbitrarily, and the final formulæ are independent of this choice), and, for any holomorphic differential, $\omega \in H^0(\mathcal{K})$ define

$$\omega(\mathscr{T}) := \mathrm{diag}(\omega(t_1), \ldots, \omega(t_{ng})). \tag{32}$$

Define the Brill–Noether-Tyurin matrix by

$$\mathbb{T} = \left[\omega_1(\mathscr{T})\mathbf{H}^t | \cdots | \omega_g(\mathscr{T})\mathbf{H}^t\right] \in \mathrm{Mat}_{ng\times ng} \tag{33}$$

Then $\mathrm{corank}\mathbb{T} = h^1(\mathscr{E})$; namely the non-Abelian theta divisor is simply the locus $\det \mathbb{T} = 0$.

In [101] we show how to use the non-abelian Cauchy kernel to construct explicitly Higgs fields and how the regular part of the Cauchy kernel in the expansion along the diagonal $p = q$ provides a natural flat connection on $\mathscr{E}$ that depends holomorphically on the moduli (outside of the non-abelian Theta divisor).

2.2 Publications

2.2.1 Published refereed papers

2021

1. M. Bertola, D. Korotkin, "Tau functions and Monodromy Symplectomorphisms" accepted Comm. Math. Phys. doi.org/10.1007/s00220-021-04224-6
2. M.Bertola, D. Korotkin, "Extended Goldman symplectic structure in Fock-Goncharov coordinates", Journal of Differential Geometry, accepted March 24 2021.
3. M. Bertola, A. Katsevich, A. Tovbis, "On the spectral properties of the Hilbert transform operator on multi-intervals", Journal of Spectral Theory, (2021), accepted September 24, 2021.
4. A. Katsevich, M. Bertola, A. Tovbis, "Inversion formula and range conditions for a linear system related with the multi-interval finite Hilbert transform in L^2", Mathematische Nachtrischten, **8**, no. 294 (2021), 1523–1546. doi.org/10.1002/mana.201800567
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6. M. Bertola; "Padé approximants on Riemann surfaces and KP tau functions". Anal. Math. Phys. 11 (2021), no. 4, Paper No. 149. doi.org/10.1007/s13324-021-00585-2
7. M. Bertola, D. Korotkin; "Spaces of Abelian Differentials and Hitchin's Spectral Covers". Int. Math. Res. Not. IMRN 2021, no. 15, 11246–11269. doi.org/10.1093/imrn/rnz142
8. M. Bertola, B. Dubrovin, Di Yang; "Simple Lie algebras, Drinfeld-Sokolov hierarchies, and multi-point correlation functions". Mosc. Math. J. 21 (2021), no. 2, 233–270.
9. M. Bertola, D. Korotkin; "WKB expansion for a Yang-Yang generating function and the Bergman tau function". (Russian) Teoret. Mat. Fiz. 206 (2021), no. 3, 295–338. doi.org/10.4213/tmf9834
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84. M. Bertola, "Bilinear semiclassical moment functionals and their integral representation" J. Approx. Theory **121** (2003), no. 1, pag. 71–99. doi:10.1016/S0021-9045(02)00044-8
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88. M. Bertola, D. Gouthier, "Lie Triple Systems and Warped Products", Rend. Mat. Appl. (7) **21** (2001), no. 1-4, pag. 275–293, Roma.
89. M. Bertola, D. Gouthier, "Warped products with special Riemannian curvature", Bol. Soc. Brasil. Mat. (N.S.) **32** (2001), no. 1, pag. 45–62. doi:10.1007/BF01238957

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90. M. Bertola, "Frobenius manifold structure on orbit space of Jacobi group; Part II", Differential Geom. Appl. **13** (3) (2000), pag. 213–233 doi:10.1016/S0926-2245(00)00027-9
91. M. Bertola, "Frobenius manifold structure on orbit space of Jacobi group; Part I", Differential Geom. Appl. **13** (2000), pag. 19–41. doi:10.1016/S0926-2245(00)00026-7
92. M. Bertola, J. Bros, U. Moschella and R. Schaeffer, "A general construction of conformal field theories from scalar anti-de Sitter quantum field theories", Nuclear Phys. B **587** (2000), pag. 619–644. doi:10.1016/S0550-3213(00)00463-6
93. M. Bertola, J. Bros, V. Gorini, U. Moschella, R. Schaeffer, "Decomposing Quantum Fields on Branes", Nuclear Phys. B **581** (2000), pag. 575-603. doi:10.1016/S0550-3213(00)00280-7

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94. M. Bertola, V. Gorini, U. Moschella, R. Schaeffer, "Correspondence between Minkowski and de Sitter Quantum Field Theory", Phys. Lett. B **462** (1999), pag. 249–253. doi:10.1016/S0370-2693(99)00927-2

2.2.2 Refereed proceedings

95. M. Bertola, A. Assra, "Random Matrix Approach for the Capacity of Large-Scale MIMO Systems using the Harish-Chandra Formula", 2020 14th International Conference on Signal Processing and Communication Systems, ICSPCS 2020 - Proceedings, doi:10.1109/ICSPCS50536.2020.9310003

96. S.T. Ali, M. Bertola, "Symplectic geometry of the Wigner transform on noncompact symmetric spaces", Inst. Phys. Conf. Ser. No. 173: Section 7 (2003), pag. 847–854.

97. H. De Guise, M. Bertola, "Coherent-state realizations of $su(n+1)$ in terms of subgroup functions", Inst. Phys. Conf. Ser. No. 173, Section 7 (2003), pag. 523–526.

2.2.3 Nonrefereed Contributions

98. M. Bertola, "Jacobi Groups, Jacobi Forms and their Applications", in "Isomonodromic deformations and Applications in Physics", C.R.M. Proceedings & Lecture Note Series (2000) J. Harnad and A. Its ed., pag. 99–111.

99. Schaeffer, U. Moschella, Marco Bertola and Vittorio Gorini, "Generation of primordial fluctuation in curved spaces", Gravit. Cosmol., Vol. **4** (1998), No. 2 (14), pag. 121–127.

2.2.4 Published books

100. Special issue of *Journal of Physics A: Mathematical and General*, Vol. **39**, No. 38 (July 2006), special issue "Random matrices and integrable systems". Guest editors: M. Bertola, J. Harnad. doi:10.1088/0305-4470/39/28/E01

2.2.5 Articles submitted

101. M. Bertola, C. Norton, G. Ruzza, "Higgs Fields, non-abelian Cauchy kernels and the Goldman symplectic structure", arXiv:2102.09520

102. M. Bertola, S. Tarricone, "Stokes manifolds and cluster algebras", arXiv:2104.13784

103. M. Bertola, "Abelianization of Matrix Orthogonal Polynomials", arXiv:2107.12998

104. M. Bertola, "Nonlinear steepest descent approach to orthogonality on elliptic curves", arXiv:2108.11576

2.3 Conference organization

1. Co-organizer, "Tau Functions, Correlations Functions and Applications", Aug 30–Sept 3, 2021, Simons Center for Geometry and Physics, NY (online because of pandemic).
2. Co-organizer, "Integrable Systems in Geometry and Mathematical Physics. Conference in Memory of Boris Dubrovin", June 28-July 2, 2021, SISSA Trieste (online because of pandemic).
3. Co-organizer, "Integrability and Randomness in Mathematical Physics", M. Bertola, G. Falqui, T. Grava, K. McLaughlin, P. Suret, 24-29 June 2019.
4. Co-organizer, 5-day workwhop "Tau Functions of Integrable Systems and Their Applications", M. Bertola, A. Its, D. Korotkin, BIRS, Banff, September 2-6, 2018
5. Co-organizer, session "Integrable Systems", M. Bertola, J-M. Maillet, ICMP 2018, Montréal, July 23-28, 2018.
6. Co-organizer of the mini-symposium "Riemann-Hilbert method and its applications in approximation theory and beyond", M. Bertola, A. Tovbis, Ubeda (Spain), July 8-13, 2018.
7. Co-organizer of the workshop " Geometry of Integrable systems", M. Bertola, T. Grava, G. Ruzza org., SISSA, Trieste, July 7-9 2017.
8. Scientific Committee member of the workshop "Asymptotic and computational aspects of complex differential equations", Centro *Ennio de Giorgi*, at *Scuola Normale Superiore*, Pisa, Italy, February 13–17 2017.
9. Co-organizer of the workshop " Asymptotics in integrable systems, random matrices, random processes and universality; in honour of Percy Deift 70th birthday ",J. Baik, M. B. , T. Kriecherbauer, K. T-R. McLaughlin, A. Tovbis, CRM, June 7-11, 2015.
10. Co-organizer of the workshop "Positive Grassmannians: Applications to integrable systems and super Yang-Mills scattering amplitudes", M. B. , J. Harnad, M. Gekhtman, CRM, July 27-31, 2015.
11. Co-organizer of the workshop "Random Matrices, Inverse Spectral Methods and Asymptotics", E. Basor, M.B., B. Eynard, A. Its, K.T-R. McLaughlin, BIRS, Banff, Oct.10-15, 2008, Web-site link
12. Co-organizer of the workshop "Random matrices, related topics and applications", E. Basor, M.B., B. Eynard, A. Its, K.T-R. McLaughlin, CRM, Montréal, Aug. 25-30 2008, Web-site link.
13. "Short program on Moduli spaces of Riemann surfaces and related topics", M. Bertola and D. Korotkin org., Centre de recherche mathématiques, Montréal, June 4–15, 2007. Web-site link

2.4 Conference participation (most recent only)

Invited speaker, "MoSCATR VII", Moscow (online), June 2021.

Invited speaker, "VIRTUAL SEMINAR SERIES: University of Glasgow, June 2020.

Researcher, "Holomorphic Differential in Mathematics and Physics", MSRI, Berkeley, October-December 2019.

Invited Speaker, "Painlevé Equations in the Midwest", MCAIM, 2019, Ann Arbor, Aug 23–24 2019. P. Miller and G. de Silva org.

Invited Speaker, "Recent Advances in Applied Integrable Systems: Theory and Computations", ICIAM 2019, Valencia, July 17, 2019.

Speaker and organizer, "Integrability and Randomness in Mathematical Physics, June 24-29, 2019, CIRM, Luminy, France.

Invited speaker Tsinghua University (Beijing), August 15 2018.

Invited researcher Fudan University (Shanghai), August 3-19 2018.

Invited Speaker "Riemann-Hilbert method and its applications in approximation theory and beyond", M. Bertola, A. Tovbis, Ubeda (Spain), July 8-13, 2018.

Invited Speaker "Hamiltonian PDEs: PDEs: Models and Applications", University of Milano-Bicocca, 25-27 June 2018.

Invited Speaker "ENS de Lyon meets SISSA. Lyon, December 5-6 2017.

Invited Speaker "Special Functions and Orthogonal Polynomials", University of Barcelona, FoCM 2017, July 16–19, 2017.

Speaker XXXVI Workshop on Geometric Methods in Physics, Bialowieza (Poland), July 2017.

Invited Lecturer "Asymptotic and computational aspects of complex differential equations", Centro *Ennio de Giorgi*, at *Scuola Normale Superiore*, Pisa, Italy, February 13–17 2017.

Invited Lecturer "Congreso de la Real Sociedad Matemática Española", Session on "Special functions, orthogonal polynomials, and applications", Zaragoza, Spain, January 30-February 3 2017.

Invited Lecturer "Special Session on Integrable Systems and Soliton Equations", Denver, October 7-9, 2016.

Invited Lecturer "Painleve Equations and Discrete Dynamics", BIRS, Banff, October 2-7, 2016.

Invited Lecturer "Infinite Analysis 16 Summer School", three hour minicourse, Nagoya, Aug. 29-Sep 3, 2016, org. R. Inoue, A. Kuniba, M. Okado, T. Nakanishi, Y. Takeyama

Invited key speaker Workshop "Random Product Matrices", Bielefeld, Aug. 22-26, 2016, org. P. J. Forrester, M. Kieburg, R. Speicher.

Invited Lecturer:"KIAS workshop on Integrable Systems and related topics", three hour minicourse, June 22-26 2016, Seoul, South Korea.

Invited speaker: "Moduli, Integrability and Dynamics", May 30-June 2, 2016, org. Takhtajan, P. Zograf, Mittag-Leffler Institute, Sweden.

Invited speaker: "Moduli spaces, Integrable systems, and topological recursions", D. Korotkin org. Montréal, January 2016.

Invited speaker: "Symposium on orthogonal polynomials, special functions and applications", June 1-5, 2015, Baltimore, USA.

Invited Lecturer: "XXXIX Summer School on Mathematical Physics", September 2014, Ravello (Italy). Six day course.

Speaker: XXXIII Workshop on Geometric Methods in Physics, Bialowieza (Poland), July 2014.

Speaker: "Spring Eastern Sectional Meeting, Baltimore, Maryland. Event: Novel Developments in Tomography and Applications", Baltimore, March 2014.

Invited Speaker at IAS, Princeton:"Workshop on Non-equilibrium Dynamics and Random Matrices", November 2013.

Invited Speaker:"Advanced School and Workshop on Random Matrices and Growth Models", ICTP, Trieste, September 2013.

Speaker:"The Eighth IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena". Session on "Randomness in Integrable Systems", Athens, GA (USA), March 2013.

Invited speaker: "Aventures en Physique Mathèmathique", *Entretiens Jacques Cartier*, Lyon, November, 2012.

Invited participant: "Integrable systems, growth processes and KPZ universality", BIRS, Banff, September 2012.

Invited speaker: "Geometry, Integrability, Quantization", SISSA (Trieste), July 2012.

Participant: XXXI WGMP, Bialowieza, Poland, June 2012.

Invited speaker: "Integrable Systems", Olsztyn (Poland), June 2012

Participant: "Strong asymptotics for Cauchy Biorthogonal polynomials", Research in Teams program, BIRS, Banff, June 2012.

Invited speaker: "Integrable systems and Random Matrices in honor to Sasha Its", May 21-23, 2012

Invited speaker: "Contemporary ways of integrability", Lisbon, May 16-18, 2012.

Invited Participant: AIM workshop on "Vector equilibrium problems and their applications to random matrix models", Palo Alto, April 2012.

Invited minicourse lecturer: École de Physique des Houches, "Random Matrices & Integrable systems", 3hour minicourse, March 2012.

Invited speaker: Institut Henri Poincaré, Paris, "GranMa 2011", October 2011.

Invited speaker: Banach Center Conference, "Formal and Analytic Solutions of Differential and Difference Equations", August 2011.

Invited speaker: Euler Institute St. Petersburg, International Conference "Painevé equations and related topics", June 2011.

Invited Visitor: Korean Institute of Advanced Studies (KIAS), Dec. 11-18 2010, host: Dr. Davide Guzzetti. Minicourse on: The Riemann Hilbert approach to the asymptotic study of orthogonal polynomials and related problems.

Invited Speaker: ISM Winter School 2011 (Jan 2011).

Invited Speaker: Integrable and stochastic Laplacian growth in modern mathematical physics, BIRS, Banff, October 2010.

Speaker: SIAM Conference on Nonlinear Waves and Coherent Structures: invited spaker in the minisymposia "Nonlinear Waves in Integrable Systems" and "Recent Advances in Nonlinear Integrable Systems", August 2010.

Speaker: XXIX Workshop on Geometric Methods in Physics, June 27 - July 3 2010, Bialowieza (PL).

Invited Speaker June 7-12, 2010 Integrable Systems in Pure and Applied Mathematics, Alghero, Italy.

Colloquium speaker February 21-28, 2010, Colloquium at the Dept. of Math. University of Central Florida.

Speaker October 24-25, 2009, AMS sectional meeting, University Park.

Speaker June 28–July 4, 2009, XXVIII Workshop on geometric methods in physics, Bialowieza, Poland.

Invited speaker June 15–20, 2009, Co-Sponsored School on Integrable Systems and Scientific Computing, ICTP, Trieste, Italy

3 External Funding history

1. (Principal and only applicant) NSERC Discovery grant, 40000 CDN per annum, 2016-2022, "Integrable Systems in Geometry, Asymptotics and Inverse Problems".

2. (Principal applicant) FQRNT Projet de rechreche en équipe, "Applications of integrable systems to Riemann surfaces and moduli spaces ",J. Harnad (Concordia), B. Marco (Concordia), B. Eynard (CEA, Saclay, France), J. Hurtubise (McGill), D. Korotkin (Concordia): 48k+12k "frais indirectes", per annum (2015–2018). The amount is equally divided amongst the participants

3. (Principal applicant) FQRNT Projet de rechreche en équipe, "Matrices Aléatoires, Processus Stochastiques et Systèmes Intégrables ",J. Harnad (Concordia), B. Marco (Concordia), B. Eynard (CEA, Saclay, France), J. Hurtubise (McGill), D. Korotkin (Concordia): 53k per annum (2011–2014) plus $ 8550 equipment. The amount is equally divided amongst the participants

4. (Principal and only applicant) NSERC Discovery grant, 36500 CDN per annum, 2011-2016, "Rigorous approaches to universality results in Random Matrix Theory, Integrable Systems and nonlinear Integrable wave equations".

5. (Principal and only applicant) NSERC Discovery grant, 17784 CDN per annum, 2006-2011, "Exact and asymptotic methods in Random Matrix Theory and Integrable Systems".

6. FQRNT Projet de rechreche en équipe, "Théorie spectrale des matrices aléatoires et des déformation isomonodromiques" , J. Harnad (Concordia), B. Marco (Concordia), B. Eynard (CEA, Saclay, France), J. Hurtubise (McGill), D. Korotkin (Concordia): 45k per annum (2006–2009) plus 15930$ equipment. The amount is equally divided amongst the participants.

7. (Principal and only applicant) NSERC Discovery grant no. 261229-03, 15000 CDN per annum, 2003-2006, "Random Matrices, Semiclassical Asymptotics and Integrable Systems" .

8. (Principal and only applicant) FCAR grant. 88353, 15000 CDN per annum, 2003-2006, "La transformée de Wigner, états cohérents et théorie quantique des champs sur espaces symétriques" (see referee reports in the Service Dossier).

9. FCAR collaborative grant 88582,"Développements en géométrie symplectique et applications à la physique mathématique": one eighth of 50000 CDN (2004), 44500 CDN (2005), 44500 CDN (2006).

3.1 Students' supervision

- Ramtin Sasani (PhD); "Symplectic geometry of inverse spectral problems" (ending 2024)
- Malik Balogoun (PhD); "Aspects of orthogonality on Riemann surfaces" (ending 2024)
- Sofia Tarricone (PhD); "The Painlevé II hierarchy: geometry and applictations" (defended Oct 2021).
- Fatane Modasheramini (PhD); "Quantization of Calogero–Painlevé system and Multi-particle quantum Painlevé equations II–VI", (April 2021);

- Maria De Martino (MsC); "Fredholm Determinant and Stochastic Processes" (Defended July 2020),
- José Gustavo Elias Rebelo (PhD); "Painlev IV equation, Fredholm Determinant and Double-Scaling Limits" (Defended April 2017; SISSA).
- Eduardo Chavez Heredia (PhD); "Shapiro-Tater conjecture and orthogonal polynomials" (since Jan 2020)
- Giulio Ruzza (PhD); "Generating functions of intersection numbers in moduli spaces" (Sept '16-Sept '19), SISSA);
- Harini Desiraju (PhD); "Fredholm determinant representations for the second Painlevé equation" (Sept '17, SISSA);
- Ayman Assra (MsC); "Random Matrices in telecommunication systems" (since Sept '16).
- Manuela Girotti; thesis on "Riemann-Hilbert approach to Gap Probabilities of Determinantal Point Processes", defended August 2014.
- Yulia Klochko: PhD. thesis on "Genus one polyhedral surfaces, spaces of quadratic differentials on tori and determinants of Laplacians", defended May 2009.
- Eva Rifkah: MsC. co-supervision, "Anisotropic Diffusion: Derivations and Parameter Adaptation for Image Noise Reduction", 2009.

3.2 Support of Post-Doctoral Fellows

- Dr. Semeon Artamonov, 10k, 2021 to date.
- Dr. Fabrizio Del Monte, 10k, 2020 to date.
- Dr. Boris Runov, 20k, 2019.
- Dr. Chaya Norton, 20k p.a., 2014- 2018.
- Dr. Janosch Ortmann, 8k p.a., 2015 to 2017.
- Dr. Thomas Bothner, 10kp.a. 2013 to 2015.
- Dr. Fedor Soloviev, 4k, 2014
- Dr. Eungyun Lee, NSERC, 10k p.a., 2012 to 2014.
- Dr. M. Cafasso, NSERC/CRM-ISM: 9k p.a., 2009–2011.
- Dr. Seung Yeop Lee, NSERC/CRM-ISM: 5k p.a., 2007–2009.
- Dr. Aleix Prats-Ferrer, FQRNT: 8k p.a., 2007–2010.
- Dr. Iana Anguelova CRM-ISM PDF: 10k for 2006–2007.
- Dr. Andrew McIntyre CRM-ISM PDF: 3k for 2005–2006 and 3k for 2006–2007.
- Dr. Mo Man Yue (CRM-ISM PDF: 4k for 2004–2005 and 10k for 2005–2006
- Dr. Gabor Pusztai (Concordia-CRM PDF : 4k) for 2003-2005

3.3 Service to the University

- NSERC Doctoral Committee, 2018.
- Putnam competition local organizer; 2004 to date.
- Academic Advisor: Sept. 2017 to date.
- External referee, PhD defence, Miguel Cutimanco, Sherbrooke University (Jan. 2021)
- External examiner, PhD defence Jack Araz (June 2020; Phys. M. Frank supervisor).
- Supervisor, MsC, Maria de Martino (June 2020; Trieste)
- Supervisor, Honours, Agnese Mantione (Kdv) (June 2020; Trieste)
- External examiner, PhD defence Nadeem Ashraf (Jan 2020; ECE A. Sebak A. A. Kishk supervisors).
- Jan 30, 2019, Comprehensive exam PhD.stud. S.Gupta (A. Sebak) ECE Concordia
- Feb 21 2019, PhD Proposal N. Ashraf (A. Sebak) ECE Concordia.
- External examiner, PhD defence Reza Movahedinia (Nov 2018; ENCS).
- Committee member, MsC. Thesis Chana Pevzner (Jul 2018; MathStat).
- External reviewer, PhD defence of Sébastien Bertrand (Sept 2017, UdeM).
- Several times committee member for evaluation of comprehensive in ENCS department.