1. Evaluate the limit if it exists, or explain why the limit does not exist.

   a. $\lim\limits_{x \to 2}(x^2 - 4)$

   b. $\lim\limits_{x \to \infty} \dfrac{x^3}{(x+1)^2}$

   c. $\lim\limits_{x \to 1^-} \dfrac{|x-1|}{x-1}$

2. Find the derivatives of the following functions.

   a. $y = (x^3 + 7x^2 - x + 1)^4$

   b. $y = \dfrac{\log_2 x}{1+x^2}$

   c. $y = (1 + e^x)\ln(2x + 4)$

3. Find $dh$ if $h = x^{1.5}$, $x = 4$, and the change in the $x$ is 0.1.

4. The total cost (in dollars) of producing $x$ HDTVs is:

$$C(x) = 10{,}000 + 200x - \frac{1}{10}x^2$$

   a. Find the total cost and the marginal cost at a production level of 100 TV's.

   b. Use the marginal cost to approximate the cost of producing the $101^{st}$ TV.

   c. Find the exact cost of producing the $101^{st}$ TV.

5. Given the price-demand equation:

$$p = 60 - 0.02x$$

   a. Find the elasticity of demand, $E(p)$.

   b. Find the values of p for which the demand is elastic, inelastic and unit elastic.

   c. How will the revenue be affected by each of the following scenarios ?

      i. If the price increases and the demand is inelastic.

      ii. If the price decreases and the demand is elastic.

      iii. If the demand is unit elastic.

6. Let $f(x) = x^4 - 2x^3$.

    a. Find the intervals where $f(x)$ is increasing and where it is decreasing. Also, find the critical points (if any) and state what each represent.

    b. Find the intervals where $f(x)$ is concave upward and where it is concave downward. Also, find the inflection points (if any).

    c. Using the results parts a. & b. above, sketch the graph.
    *Hint*| Feel free to use the domain of $f(x)$ as well as its intercepts in your sketch.

7. Compute the following anti-derivatives:

    a. $\int 2x(x^2 + 4)^3 \, dx$

    b. $\int \frac{x^2 e^x - 2x}{x^2} \, dx$

8. Suppose that a country has a Lorentz curve of the form $f(x) = x^a$ and a Gini Index of $0.268$. Find $a$.

**NOTE** [**REFERENCES**]:

Some questions in this document have been selected from final exams and midterms at Concordia University.

**ANSWER KEY:**

1.

| | |
|---|---|
| a. | 0 |
| b. | $\infty$ |
| c. | -1 |

2.

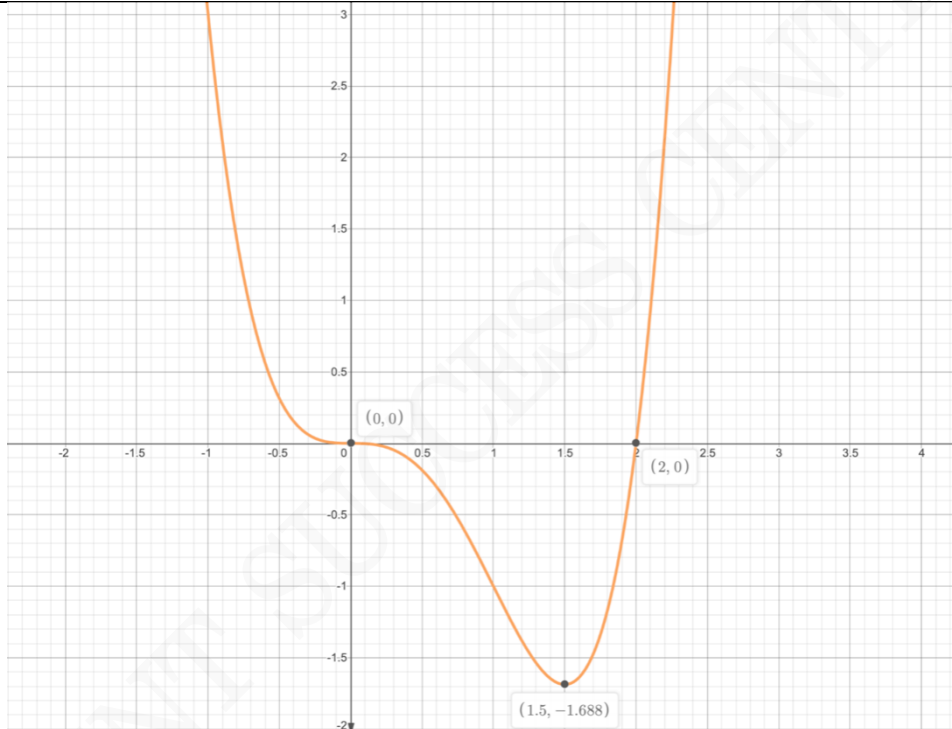| | |
|---|---|
| a. | $\dfrac{dy}{dx} = 4(x^3 + 7x^2 - x + 1)^3(3x^2 + 14x - 1)$ |
| b. | $\dfrac{dy}{dx} = \dfrac{(1 + x^2)\left(\dfrac{1}{x \ln 2}\right) - 2x \log_2 x}{(1 + x^2)^2}$ |
| c. | $\dfrac{dy}{dx} = e^x \ln(2x + 4) + \dfrac{1 + e^x}{x + 2}$ |

3. $dh = 0.3$

4.

| | |
|---|---|
| a. | $C(100) = \$29000$ , $C'(100) = \$180$ |
| b. | $Approximate\ cost\ = C'(100)\ = \$180$ |
| c. | $Exact\ cost\ = C(101) - C(100) = \$179.9$ |

5.

| | |
|---|---|
| a. | $E(p) = \dfrac{p}{60 - p}$ |
| b. | **inelastic**: $\quad 0 < p < 30$<br>**unit elastic**: $\quad p = 30$<br>**elastic**: $\quad 30 < p < 60$ |
| c. | i.  Revenue will increase<br>ii.  Revenue will increase<br>iii.  It will not affect revenue |

6.

| | |
|---|---|
| a. | $increasing$: $\left(\frac{3}{2}, \infty\right)$ ; $decreasing$: $(-\infty, 0), \left(0, \frac{3}{2}\right)$<br><br>$critical\ points$: $at \rightarrow x = 0\ (saddle)$<br>$\rightarrow x = \frac{3}{2}\ (where\ local\ minimum\ occurs)$ |
| b. | $concave\ up$: $(-\infty, 0), (1, \infty)$ ; $concave\ down$: $(0,1)$<br><br>$inflection\ points$: $at\ \ x = 0\ \ and\ \ x = 1$ |
| c. |  |

7.

| | |
|---|---|
| a. | $ans = \frac{1}{4}(x^2 + 4)^4 + C$ |
| b. | $ans = e^x - 2ln|x| + C$ |

8. $a = 1.732$