Department of Mathematics & Statistics
Concordia University

MAST 234

Linear Algebra and Applications I

Winter 2025

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When sending me an email, please include "MAST 234" in the subject line.

Class Schedule: Tuesdays and Thursdays, 10:15-11:30.
Note: There will be a mid-term break from February 24 to March 2.

Office hours: TBA.

Text: There is no mandatory textbook for this course, all the material will be available in the Lecture files and the Classwork files that will be posted on the course page on Moodle. For additional reading and practicing at home most of the topics learned in this course can be found in the following complementary open-source text:

(A) *Linear Algebra with Applications*, by W. Keith Nicholson, Open Texts by Lyryx
https://lyryx.com/wp-content/uploads/2018/01/Nicholson-OpenLAWA-2018A.pdf

Prerequisites: MATH 204 or equivalent is a prerequisite for this course.

Objectives: There are two major concepts, *General Vector Spaces* and *Linear Transformations,* on which this course is based. In learning these concepts we will use related constructs such as *vectors*, *matrices,* and *systems of linear equations*. The objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further Linear Algebra courses in your curriculum.

Pedagogy: Classes are interactive and start with a lecture introducing the principal concepts of the topic considered that day, followed by problem solving by students in the lab equipped with computers. Mathematical issues that arise during problem solving are discussed in class.

Software: All coursework will be carried out using *SageMath*, using *JupyterLab* as an IDE. Both *SageMath* and *JupyterLab* are free, open source, software systems. If you would like to install these programs on your personal computer, you can visit https://www.sagemath.org/ and https://jupyter.org/, or ask your instructor for help. In this course the software is only used as a computational tool, ***not* as**

an object of study in itself. All assignments, quizzes, the midterm test and the final examination are done using SageMath.

Assignments: Assignments (home work) are given and submitted online through Moodle. Late assignments **will not** be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). *W*orking regularly on the assignments, class attendance and working on the problems in the class, is essential for success in this course.

Midterm Test: There will be **one midterm test**, written in class on **Tuesday March 4 2025**, and will be based on the material of weeks one through 6. The midterm test is closed-book and written in the class using SageMath. It will contribute up to 30% to your final grade (see the Grading Scheme).

NOTE: It is the Department's policy that tests missed for any reason, **including illness,** cannot be made up. If you missed the midterm for a valid reason, the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and the quizzes.

Final Exam: The Final Examination will be 3 hours long (**closed-book** exam, no notes or electronic material is allowed) written using SageMath in the class equipped with computers. Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts with the schedule of the final exam must be reported directly to the Examinations Office, **not** to the Instructor. **Students are to be available until the end of the final exam period.** Conflicts due to travel plans **will not** be accommodated.

NOTE: There are **no supplemental exams** for this course.

Grading Scheme: The final grade will be based on the higher of (a) and (b) below:

(a) 15% for the assignments
30% for the class test
55% for the final examination

(b) 15% for the assignments
10% for the class test
75% for the final examination

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

Disclaimer: The instructor reserves the right to make changes to the course outline and course content should this be necessary for academic or other reasons.

CONTENTS

Week	Lectures TOPICS	Sub-Topics considered	Complementary Reading (KN Text)

1	Linear Systems
	Row Equivalent Matrices &
	GJ Method for LS
		• Review of systems of linear equations.
		  ○ Matrix form of a system
		  ○ Matrix of the system; augmented matrix
		  ○ Elementary Row Operations
		  ○ Row Echelon Form
		  ○ GJ Solutions for a system of linear equations
			Sections
			KN: 1.1, 1.2

2	Span of Vectors
	Matrices
	Consistent systems
	RANK
		• Review of vectors and matrices
		  ○ Vectors in R^n. Linear combination of vectors
		  ○ Matrix Operations, Matrix-vector products
		• Span of a set of vectors
		• Matrix Rank, Column & Row Spaces
		• Rank Theorems, Consistency of systems Ax=b
			Sections
			KN: 1.2, 2.1, 2.2, 2.3

3	Homogeneous Systems Linear Dependence	• Linear dependence of vectors/vector sets • Homogeneous Systems of Equations • Null Space of a matrix	**Sections** **KN: 1.3, 5.2, 5.4**

4	Operations on Matrices
	Determinants
		• Operations on matrices
		  ○ Multiplication of matrices: definition and properties
		  ○ Special matrices
		• Determinants (an introductory overview)
			Sections
			KN: 2.3, 2.4, 3.1, 3.2, 5.4

5	Matrix Inverse	• Left and Right inverses of Matrices. • Invertible matrix (square) • Properties of invertible matrices.	**Sections** **KN: 2.4, 3.2**

6	Linear Transformations in R^n	• Linear transformations in R^n
		  ○ Domain, Co-domain, Range:
		  ○ The Linearity properties: definition and examples
		  ○ Matrices and linear maps: Theorems 1-3.
		  ○ Composition of linear mappings (Theorem 2.3.8)
			Sections
			KN: 2.2, 2.3, 2.6

7	General Vector Spaces and Subspaces	**MIDTERM TEST** (based on the material of Lectures 1-6) • Examples of Vector Spaces R^n & Euclidean vector spaces Polynomials as vectors • Definition of Vector spaces, and its Properties • Linear Independence in Vector spaces. • Vector Subspaces	**Sections** **KN: 6.1,6.2, 6.3**
8	Basis, Coordinatization Transition Matrices	• Basis for a vector space & Dimension Theorems on Spans and Basis • Coordinate vector, Coordinatization • Coordinates in different basis • Transition matrices from one basis to another.	**Sections** **KN: 6.3, 6.4**
9	LINEAR MAPS (GENERAL) LINEAR MAPS &MATRICES	• Linear transformations in abstract vector spaces, other than R^n • Rank-Nullity Theorem • Matrix representation of a linear transformation • Matrices of linear mappings of a vectors space into itself ○ Similar matrices	**Sections** **KN: 9.1, 9.2, 5.5**

10	EIGENVALUES & EIGENVECTORS, EIGENTHEORY	• Eigentheory Basics o Eigenvectors and eigenvalues of matrices o Characteristic polynomial o Eigenvectors and eigenvalues of a linear operator	**Sections KN: 3.3, 5.5. 9.3**
11	DIAGONALIZATION OF SQUARE MATRICES & APPLICATIONS	• Diagonalizable matrices and diagonalizable linear operators o Conditions of diagonalizability: basis of eigenvectors. o Numerical methods for diagonalizatio • Application of diagonalization (Dynamical systems)	**Sections KN: 5.5, 9.2**
12	**REVIEW**	Review Classes	

Student Services
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Academic Integrity and the Academic Code of Conduct
This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." *[Undergraduate Calendar, Sec 17.10.2]*

Behaviour
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