



BRIEFING NOTES

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MATHEMATICAL MODELLING OF COVID-19 AND FLATTENING THE TRANSMISSION CURVE

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SUMMARY

- ✚ What is the Mathematical Model and How Does it Work for COVID-19?
- ✚ What Does COVID-19 Model Tell Us?
- ✚ How Non-Pharmaceutical Interventions Affect the Spread of the Coronavirus?
- ✚ What is the Best Public Policy for Turn Back to Normal Conditions and is it at all Feasible to Return to Normal Conditions?

Abstract

Mathematical modelling allows decision makers to examine “what-if” scenarios under different public policies. According to a developed model, a combined physical distancing policy together with a nation-wide testing, isolation, and contact tracing of confirmed cases can be an efficient public intervention policy provided that it is persistent beyond only a few months.

Mathematical Model and Purpose

- ✚ A mathematical model of COVID-19 is a set of equations that aim at simulating the epidemiologic spread of the disease based on given factors including the coronavirus reproduction rate, comorbidity state of the population that are exposed to the coronavirus, admission rate and capacity of the health care system settings, public intervention efforts, among others.
- ✚ The objective is to examine variations in certain interesting variables such as the epidemic attack rate (i.e., the percentage of population that is infected with the disease), the number of required simultaneous access to the intensive care units (ICUs) as a function of time.
- ✚ The ultimate goal would be to determine the optimum control strategy that would minimize the epidemiologic spread and the societal disruption [3], to a manageable level based on an available health care system capacity.

Background

- ✚ The set of mathematical equations that can model the objectives that are identified above are referred to as the epidemiologic system model (ESM). Many recent COVID-19 epidemiological system models that are based on the Chinese and the Canadian Community Health Survey, [1] and [2], respectively, have been published.
- ✚ The present BN aims to explain the mechanics behind the ESM derived model in [2] and present the key results that are obtained therein, and propose further changes to the model to address the limitations of the model (as a topic of future work).

Key Assumptions and Brief Mechanics of the ESM Model

The basic assumptions used in [2] have been listed in bullets items and the basic mechanics of the model is briefly provided and explained in Note-1 to Note-4 below.

- ✚ It is assumed that the recovered individuals remained immune from re-infection for the duration of the COVID-19 epidemic
- ✚ Disease transmission within the health care settings have not been included in the model.
- ✚ Variables of the ESM (the model independent variables and states) are as listed below:
 - 1- Susceptible
 - 2- Exposed
 - 3- Exposed and quarantined
 - 4- Infectious, pre-symptomatic
 - 5- Infectious, pre-symptomatic, in isolation
 - 6- Infectious - mild
 - 7- Infectious - severe
 - 8- Infectious - mild, in isolation
 - 9- Infectious - severe, in isolation
 - 10- Isolated - mild, not previously in quarantine
 - 11- Hospitalized, never in ICU
 - 12- Hospitalized, pre-ICU admission
 - 13- Hospitalized, in ICU
 - 14- Hospitalized, post-ICU
 - 15- Recovered
 - 16- Dead
- ✚ The model was stratified by 5-year age groups using the 2019 population estimates [4].
- ✚ Contacts within and between age groups were based on the POLYMOD study [5] using contact data specific to the United Kingdom
- ✚ The model was further stratified by health status to account for differential vulnerabilities to severe infection among those with underlying health conditions. Comorbidity estimates were obtained by age from the Canadian Community Health Survey (CCHS) for Ontario and included the following conditions, namely: healthy, hypertension, heart disease, asthma, stroke, diabetes and cancer [6].

Note-1: The above implies that every model variable/state (among the 16 model variables/states that are introduced above) is categorized into 5 age groups and sub-categorized further into 7 cormobidity health conditions with a total of representing 16x5x7 model variables/states.

Note-2: In other words, a model variable/state such as $x_{6_i,j}(t)$ denotes the number of non-isolated individuals within the population at time t having a mild infection in the group age i with cormobidity health condition j .

Note-3: Each model variable/state at time t_2 such as $x_{n_i,j}(t_2)$ can be estimated by a weighted sum of all the model variables/states at time t_1 , where t_2 is a day after t_1 . Hence, a motion is

generated in the model variables/states in a sense that the number of individuals in a particular variable/state change on a day by day basis, depending on the interactions between the variables/states.

Note-4: Governmental interventions are affecting the epidemiological motion of the model through certain weights corresponding to other variables/states. For instance, an effective social distancing measure generally drives a particular variable/state at t_2 less sensitive to the variables/states in t_1 .

- ✚ In absence of a vaccine, public control measures focus on the use of non-pharmaceutical interventions [6]. These non-pharmaceutical interventions include “case-based” measures such as testing, contact tracing, isolation (of infected cases) and quarantine (of exposed cases); and “non-case-based” measures such as reducing the probability of transmission given an effective contact (e.g., hand hygiene and cough etiquette) and physical distancing measures to reduce the contact rate in the population.
- ✚ The population is broadly categorized into 3 distinct classes, namely (1) Susceptible (i.e., neither exposed nor infected to COVID-19), (2) Exposed, where individuals have been in contact with infected people but have not yet manifested symptoms (asymptomatic), and (3) Infectious, where COVID-19 had been tested and positively confirmed.
- ✚ Interaction between the above listed three fundamental classes have been identified and explained in Figure-1 of [2]. Due to probabilistic nature of interactions, the model itself is also producing stochastic results in a sense that the output at each run will be different even under non-varied circumstances.
- ✚ It is considered that the COVID-19 patient would either completely recover (in quarantine or with hospital care) or yield death in ICU in most severe cases.
- ✚ Limited public health care facilities are considered.

Simulations

The model introduced above has been simulated under the following conditions:

- ✚ Base Case: No public intervention is realized
- ✚ Enhanced Detection: Sufficient amount of resources are allocated in order to timely test susceptible individuals.
- ✚ Physical Distancing: Restrictions are devised for each age group in order to reduce the probability of exposure among individuals.

For each scenario the system output includes: **(1)** Prevalent cases requiring ICU, and **(2)** Percentage of infected population.

Important Findings and Conclusions

Through simulations of the model in [2] the following findings can be observed.

- ✚ The Base-Case scenario (where no public interventions are considered) shows that 50% to 60% of population could be infected in a relatively short period of time (around 200 days)

with maximum attack rates within the age groups of 5-14 years old, until the disease is naturally faded away through herd immunity.

- ✚ Prevalent cases requiring ICU in the Base-Case scenario could exceed 0.35% of the population (in less than a year) which is 50 times greater than what is possibly administered in Canada in short term.
- ✚ Physical distancing proves to be very efficient in terms of controlling prevalent cases requiring ICU only when the policy is persisting beyond 12 months. If physical distancing is adopted for a 6 months period (for instance), the number of prevalent cases requiring ICU will be reduced to half (in comparison with the Base Case) in a dispersed period of time (that is, 300 to 600 days, instead of a year). This is still far from manageable through existing medical facilities in Canada.
- ✚ Physical distancing policy if prolonged beyond 12 months will diminish the total infected population to less than 20% after 2 years, whereas Enhanced Testing and isolation of the confirmed cases (if administered alone with no physical distancing policy) will have negligible effect in controlling the spread of the virus.
- ✚ Physical distancing policy if further prolonged beyond 16 months would be able to stop the spread of the virus.
- ✚ In order to control the epidemic through testing/isolation policy (only), the policy has to remain enforced well beyond 2 years.
- ✚ A dynamic control strategy can be advisable where a combination of physical distancing with testing and isolation of confirmed cases is devised for at least 18 months. In this control strategy physical distancing can be relaxed when testing proves that the number of infected cases is manageable. Consequently, physical distancing would be switched on and off in response to the testing outcomes.

Challenges

The main challenge for pandemic response is that in a fully susceptible population, although non-pharmaceutical interventions may slow the disease transmission while they are in place, once the intervention is lifted (or compliance with the intervention becomes low), transmission of the pathogen rebounds rapidly [2], [6], [7]. In case of COVID-19, despite what has been demonstrated through simulations it may not be possible to minimize morbidity and mortality, and societal and economic disruption at the same time.

Recommendations and Future Work

- ✚ Mathematical modelling as presented in [1] and [2] proves to be a useful tool for examination of “what-if” scenarios. The shortcomings of the model presented in this BN is in integration of control measures with model variables/states. In other words, certain cases with and without



public control measures are embedded in the model as distinct variables/states. This reduces maneuverability of the model to arbitrary public protocols.

- ✚ In case the models are reformulated with public control protocols as separate inputs, the model could be more flexible and responsive. Through collection of data from Canadian health organizations it might be feasible to optimize the weight factors defining interactions of the system states through machine learning and AI techniques. This eventually entails an online controlled architecture that optimizes control inputs in response to the testing outcomes.

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